

Testing Quantum Mechanics at High-energy Colliders

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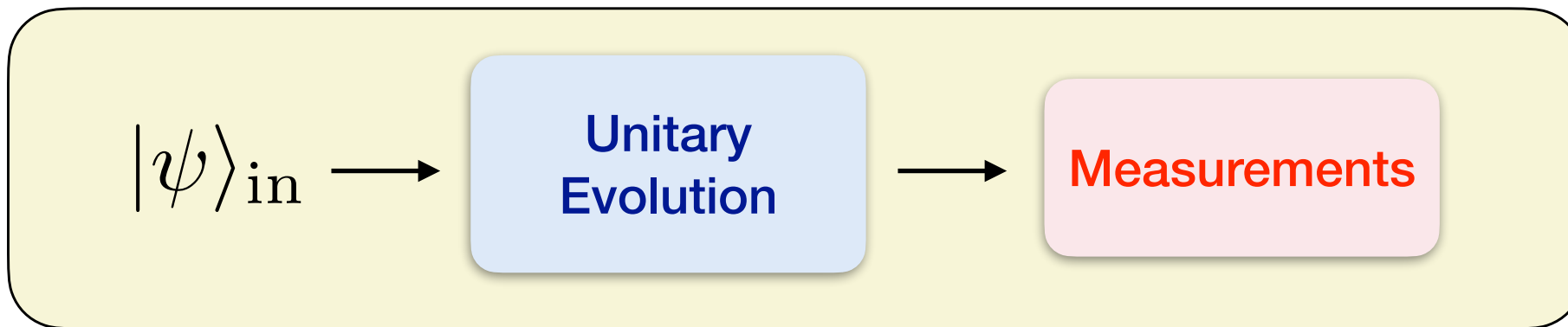
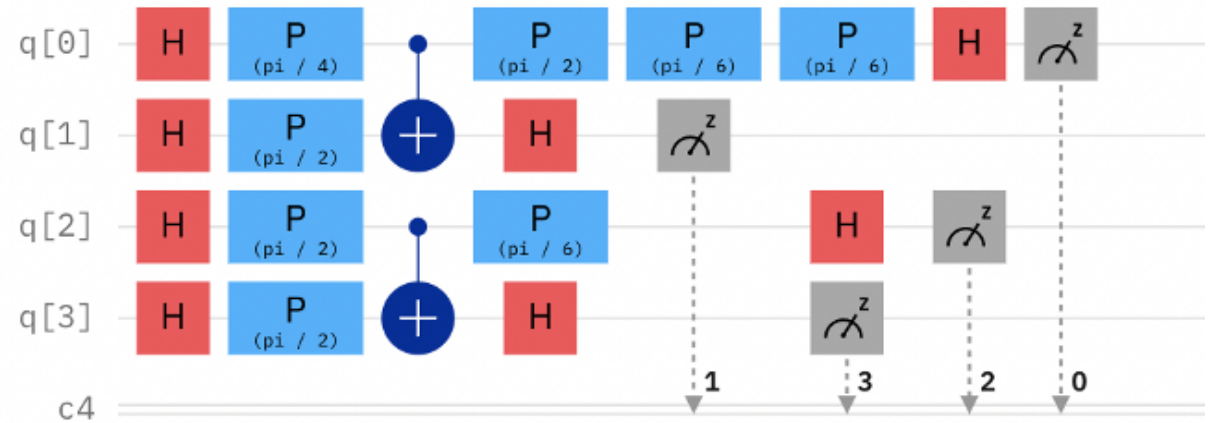
[arXiv:2412.01892 \[hep-ph\]](https://arxiv.org/abs/2412.01892)

01/09/2025 ICEPP Seminar @ Tokyo U (Hongo)

Particle Collider = Quantum Computer



C. Altomonte, A.Barr [2312.02242]



To investigate the **computational property** of colliders, we need to **control input (spin) states**.

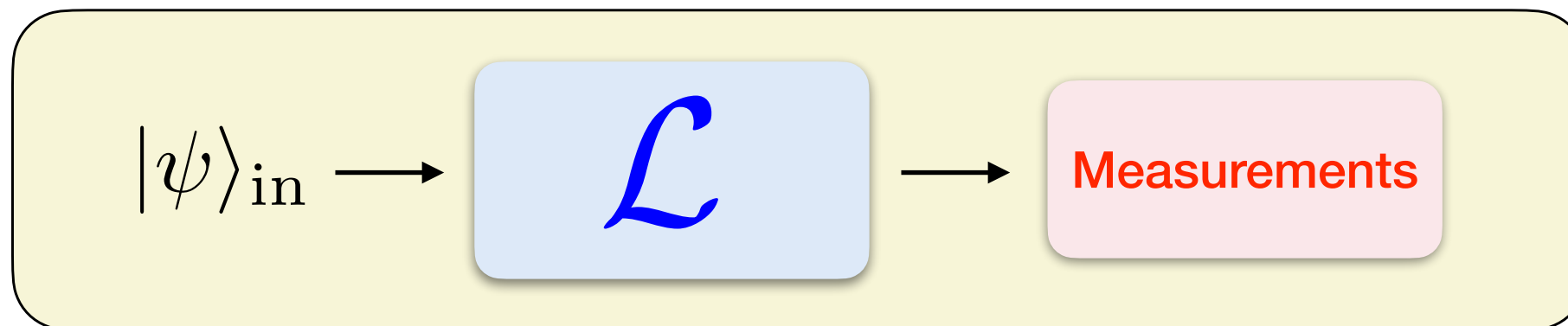
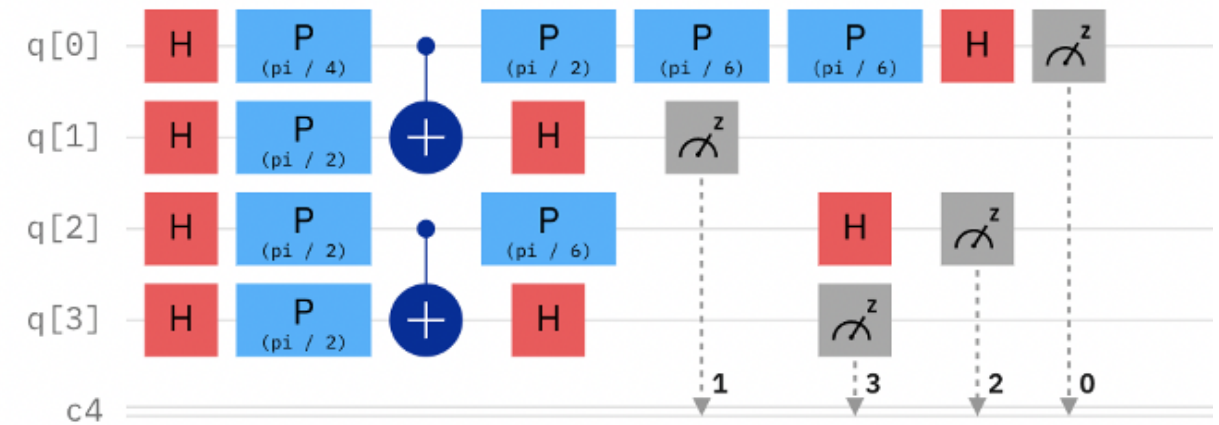
➡ **Polarised lepton colliders**

Particle Theory = Quantum Computer

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi \\ & + \chi_i y_{ij} \chi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$

=

C. Altomonte, A.Barr [2312.02242]



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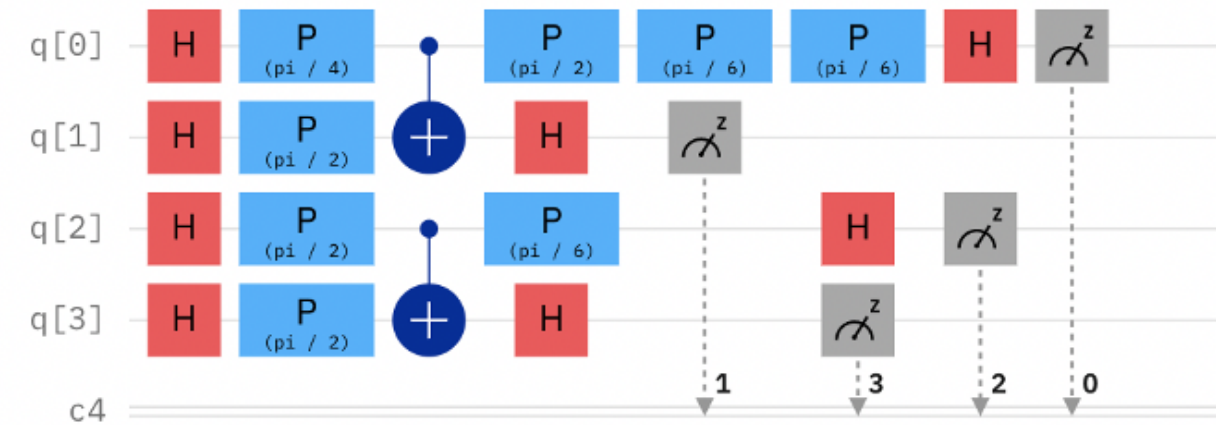
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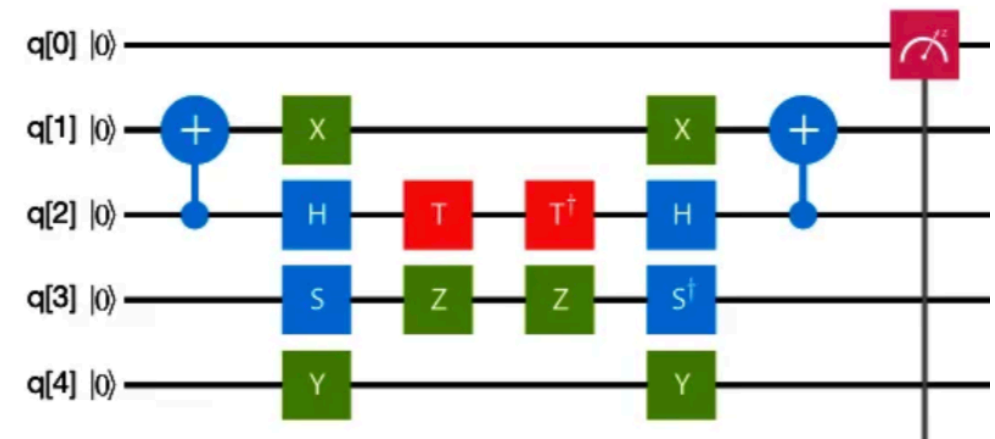
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C. Altomonte, A.Barr [2312.02242]



$$\mathcal{L}_{\text{BSM}} =$$



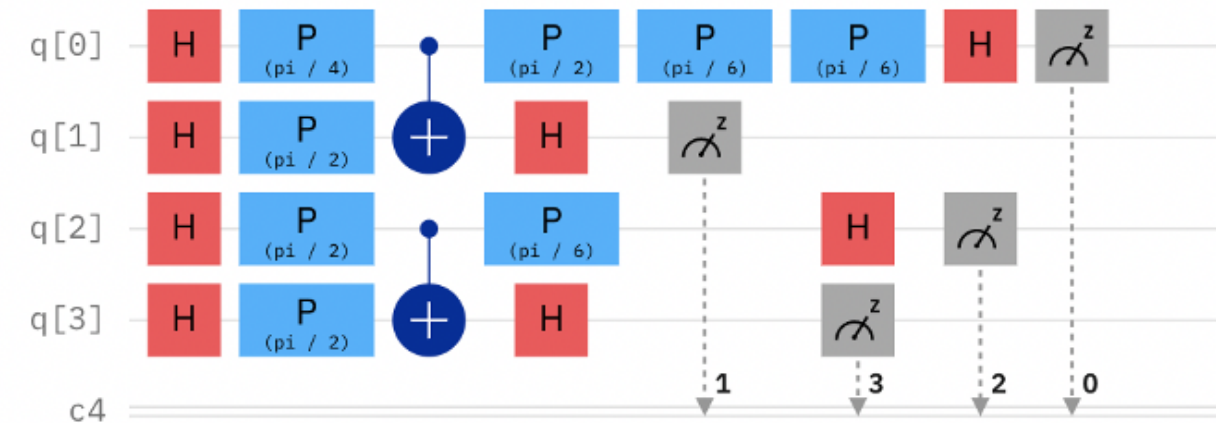
C. Altomonte, A.Barr, M.Eckstein, P.Horodecki, KS [2412.01892]

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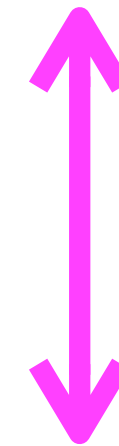
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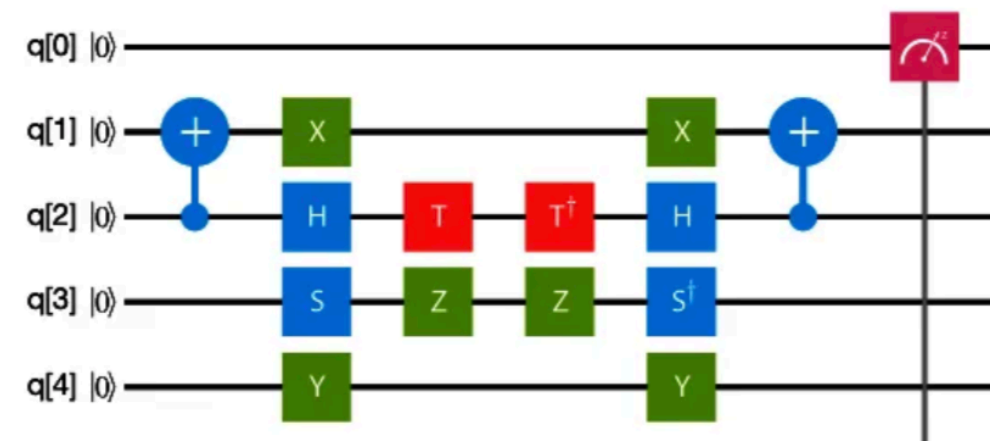
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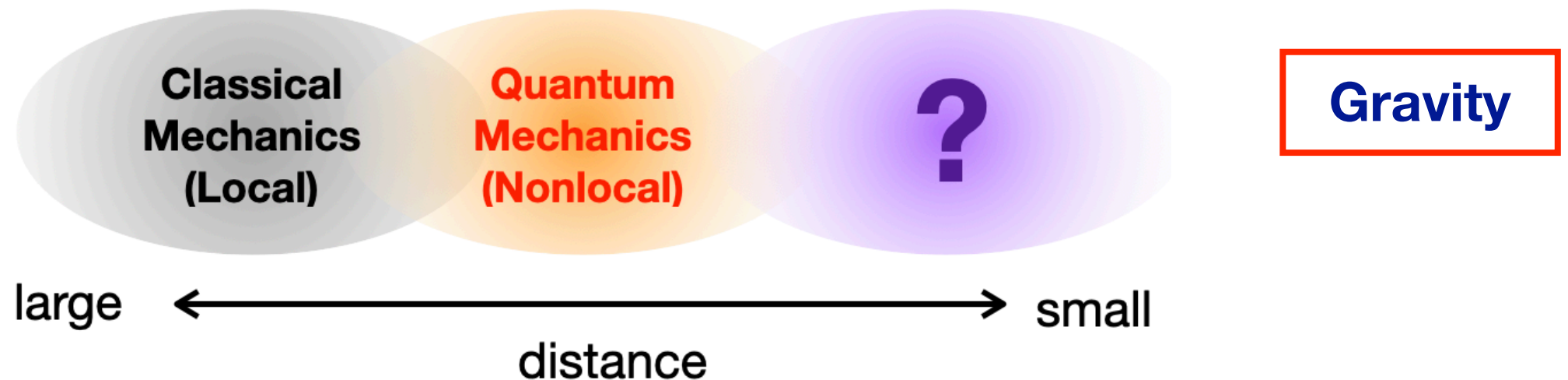
- New precision test of SM
- Foundational test of QM



$$\mathcal{L}_{\text{BSM}} =$$



High Energy Test of Quantum Mechanics



- Motivation:**
- QM might be a low energy effective theory of more fundamental short-distance theory.
 - QM might be modified at shorter distances to be married with gravitation.

Currently, **no LHC analysis** can distinguish between anomalies from **QFT-based BSM** and those from **beyond-QM**

- **Non-linear extensions of QM:** [Weinberg (1989), Polchinski (1991), D.E.Kaplan, S.Rajendran, (2021)]

$$i\partial_t|\chi\rangle = \int d^3x \left[\hat{\mathcal{H}}(x) + \langle\chi|\hat{\mathcal{O}}_1(x)|\chi\rangle\hat{\mathcal{O}}_2(x) \right] |\chi\rangle$$

non-linear state-dependent term

Plan

- Review: qubits, density matrices, quantum state tomography
- Quantum State Tomography at colliders: $pp \rightarrow t\bar{t}$
- Quantum Process Tomography (QPT), Choi matrix
- QPT of $e^+e^- \rightarrow t\bar{t}$ with polarised lepton collider
- Summary

Qubit

- The qubit system has two basis kets: $|0\rangle$ and $|1\rangle$. The general state is

$$|\Psi\rangle = c_0|0\rangle + c_1|1\rangle \quad (c_i \in \mathbb{C}, |c_0|^2 + |c_1|^2 = 1)$$

- The n -qubit system has 2^n basis kets:

$$|\psi_1\psi_2\cdots\psi_n\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle \quad |\psi_i\rangle \in \{|0\rangle, |1\rangle\}$$

Ex.) The general state in the **2-qubit** system:

$$|\Psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle \quad \left(c_{ij} \in \mathbb{C}, \sum_{i,j} |c_{ij}|^2 = 1 \right)$$

Quantum Mechanics

- Consider a measurement of an observable $\hat{A}$ on a pure state $|\psi\rangle$.

$$\hat{A} = \sum_a a|a\rangle\langle a|$$

$$\hat{A}|a\rangle = a|a\rangle \quad \langle a|a'\rangle = \delta_{aa'}$$

probability to find $a \rightarrow$ $P_{|\psi\rangle}(a) = |\langle a|\psi\rangle|^2$

post-measurement state $\rightarrow$ $|\psi\rangle_{\text{after}}^{(a)} = |a\rangle$

expectation value $\rightarrow$ $\langle \hat{A} \rangle_{|\psi\rangle} = \sum_a a P_{|\psi\rangle}(a) = \langle \psi | \hat{A} | \psi \rangle$

Density operator (matrix)

- Consider a statistical ensemble of pure states: $\{p_1; |\psi_1\rangle\}, \{p_2; |\psi_2\rangle\}, \dots$

$$p_i \geq 0, \sum_i p_i = 1$$

- We define the state is represented by the **density operator (matrix)**

$$\rho \equiv \sum_i p_i |\psi_i\rangle \langle \psi_i| \longrightarrow \rho = \rho^\dagger, \text{Tr} \rho = 1, \langle \xi | \rho | \xi \rangle \geq 0, \forall |\xi\rangle$$

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probability to find $a \rightarrow P_\rho(a) = \sum_i p_i P_{|\psi_i\rangle}(a) = \langle a | \rho | a \rangle = \text{Tr} [\Pi_a \rho]$

post-measurement state $\rightarrow \rho_{\text{after}}^{(a)} = |a\rangle \langle a| = \frac{\Pi_a \rho \Pi_a}{P_\rho(a)}$

expectation value $\rightarrow \langle \hat{A} \rangle_\rho = \sum_a a P_\rho(a) = \sum_a a \langle a | \rho | a \rangle = \text{Tr} [\hat{A} \rho]$

$$\Pi_a \equiv |a\rangle \langle a|$$



projective
measurement
operator

- If the state is pure, $\rho = |\psi_1\rangle \langle \psi_1|$, ρ has only one nonzero eigenvalue, $p_1 = 1 \rightarrow \text{Tr} \rho^2 = 1$

- If the state is mixed, $\rightarrow \frac{1}{d} \leq \text{Tr} \rho^2 < 1$

$$d \equiv \dim \mathcal{H}$$

- Maximally mixed state $\rightarrow \rho_{\text{max mix}} = \frac{1}{d} \cdot \mathbf{1}_{d \times d}$

- A classical ensemble is different from a coherent superposition.

measure z -component

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow_z\rangle + |\downarrow_z\rangle] \longrightarrow \uparrow_z (50\%), \downarrow_z (50\%)$$

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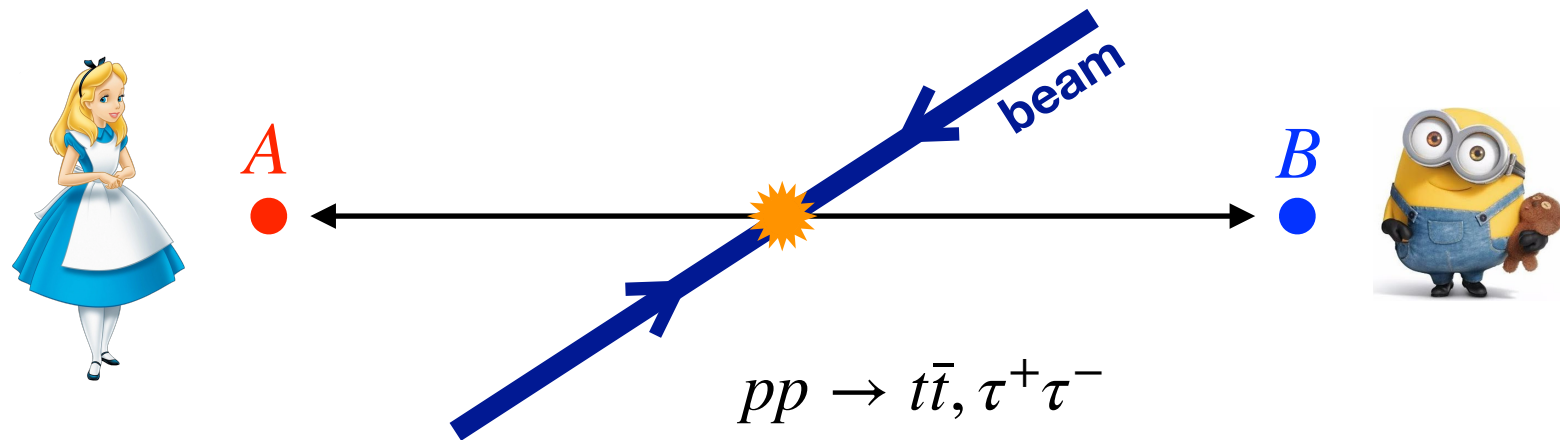
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Two-qubit system



- For a 2-qubit system, the **density matrix** can be expanded by two independent Pauli matrices:

$$\rho = \frac{1}{4} \left(\mathbf{1}_{AB} + B_i \cdot [\sigma^i \otimes \mathbf{1}_B] + \bar{B}_i \cdot [\mathbf{1}_A \otimes \sigma^i] + C_{ij} \cdot [\sigma^i \otimes \sigma^j] \right)$$

- There are 15 parameters: $B_i, \bar{B}_i, C_{ij} \in \mathbb{R} \quad (i, j = 1, 2, 3)$

$$\hat{s}_i := \sigma_i$$

$$\text{Tr}[\sigma_i \sigma_j] = 2\delta_{ij}$$

$$B_i = \text{Tr}[\rho \hat{s}_i^A] = \langle \hat{s}_i^A \rangle$$

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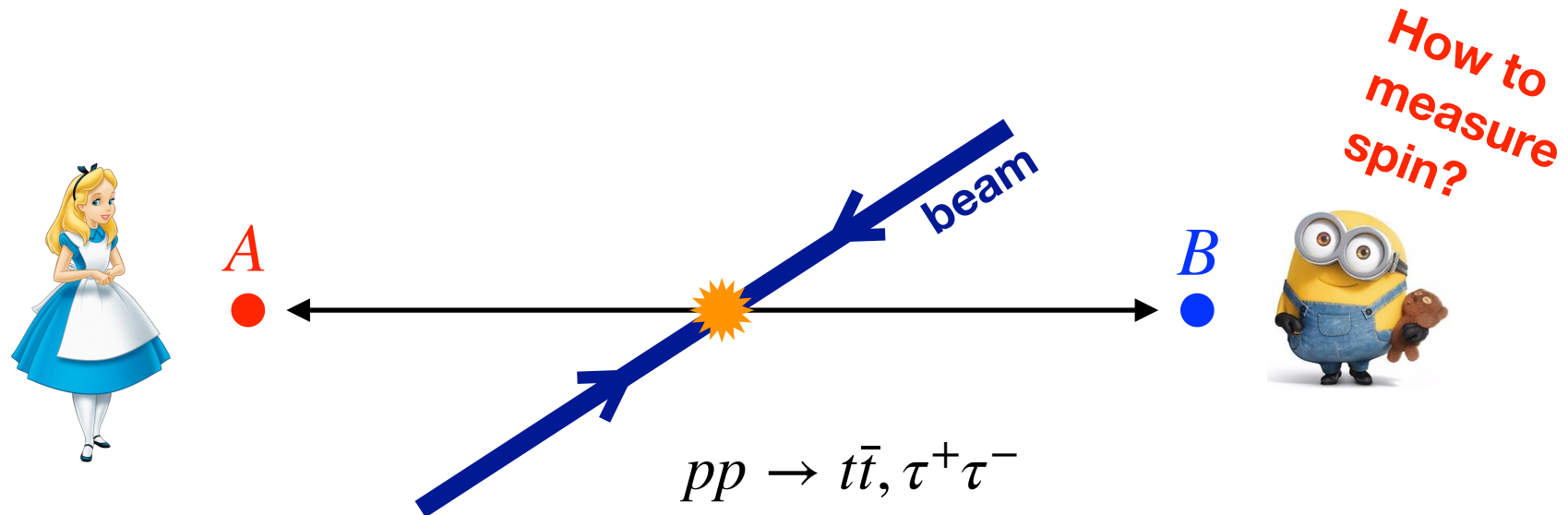
$$C_{ij} = \text{Tr}[\rho \hat{s}_i^A \hat{s}_j^B] = \langle \hat{s}_i^A \hat{s}_j^B \rangle$$

spin polarisation of A and B

spin-spin correlation

- An experimental reconstruction of the density matrix is called “**Quantum State Tomography**”

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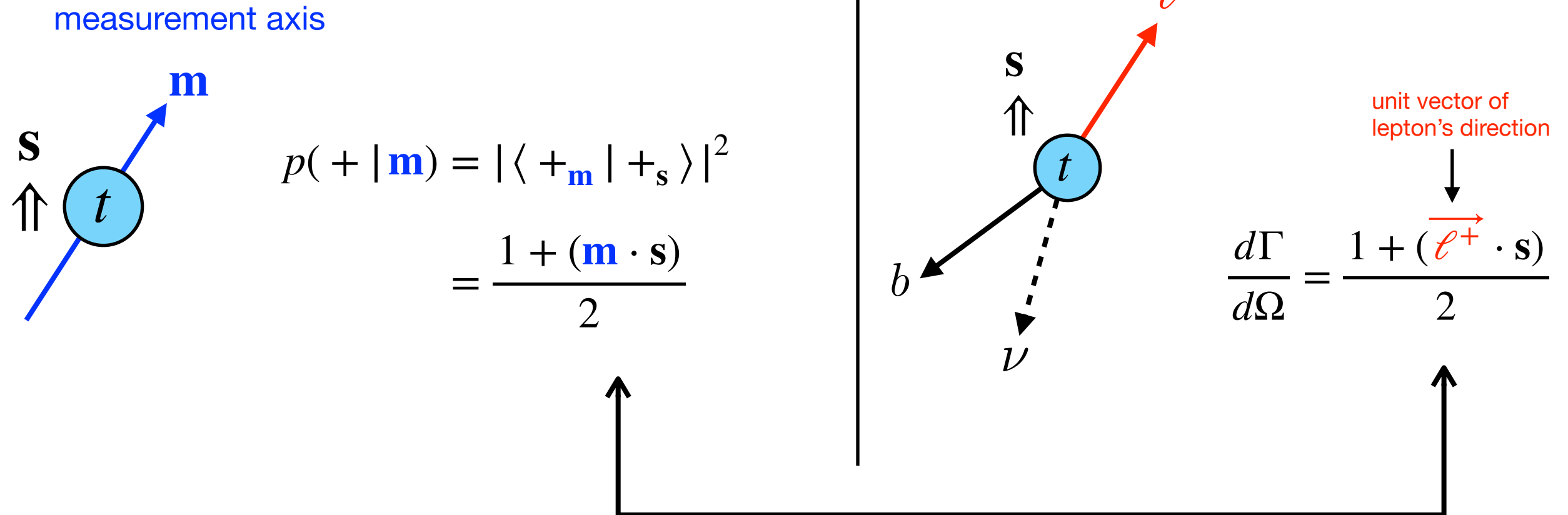
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spin-spin correlation

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Particles with chiral decays are their own polarimeters

- Consider $t \rightarrow b\nu\ell^+$ at the top's rest frame.



by measuring the direction of ℓ^+ at the top's rest frame, we can infer the spin of the top

Works for *weakly decaying particles* in the SM: τ , t , $W^\pm$, Z^0

$\mathbf{s}$: spin direction of the decaying particle
 $\mathbf{x}$: direction of decay product x (unit vector)

$$\frac{1}{\Gamma} \frac{d\Gamma}{d(\mathbf{x} \cdot \mathbf{s})} = \frac{1 + \alpha_x \cdot (\mathbf{x} \cdot \mathbf{s})}{2}$$

$\alpha_x \in [-1, +1]$: spin analyzing power

$$\tau^- \rightarrow \pi^- \nu : \alpha \simeq 1$$

$$\tau^- \rightarrow \rho^- \nu : \alpha \simeq 0.45$$

$$\tau^- \rightarrow e^- \nu \bar{\nu} : \alpha \simeq -0.33$$

$$t \rightarrow b \nu \ell^+ : \alpha \simeq 1$$

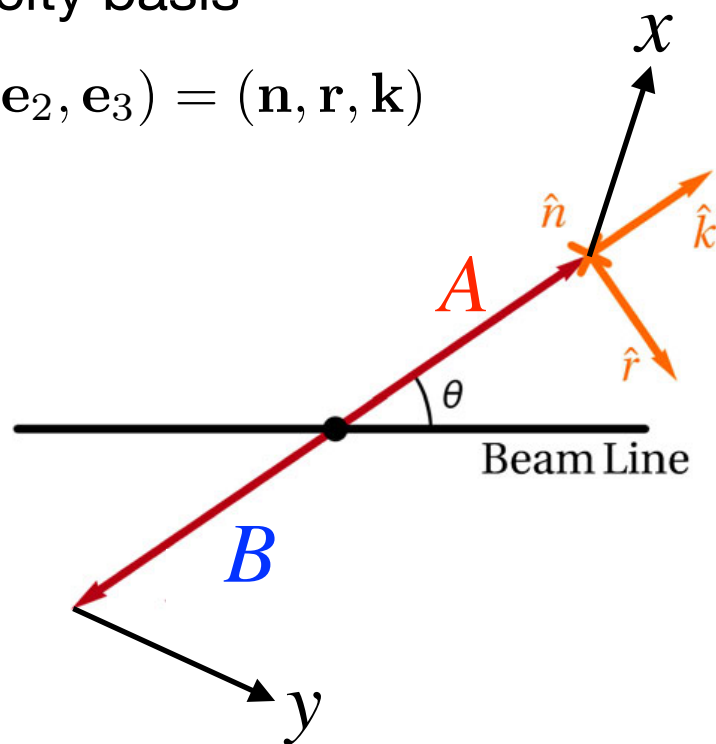
$$t \rightarrow b \nu \ell^+ : \alpha \simeq 0.39$$

$$t \rightarrow b u \bar{d} : \alpha \simeq -0.32$$

$$t \rightarrow b u \bar{d} : \alpha \simeq 0.97$$

helicity basis

$$(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) = (\mathbf{n}, \mathbf{r}, \mathbf{k})$$



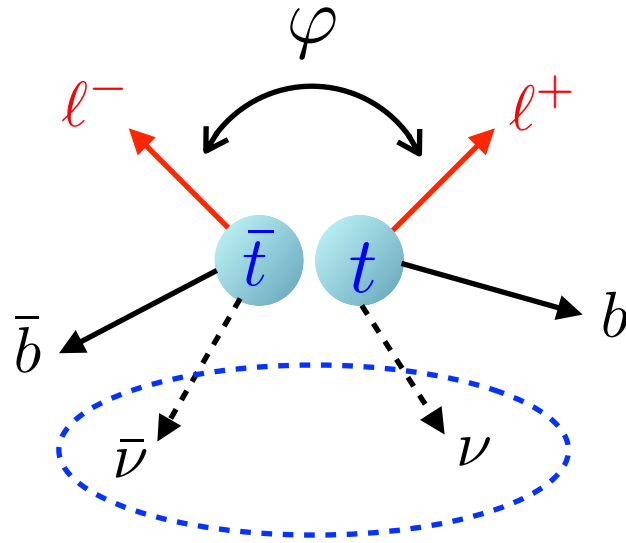
$$B_i = \langle \mathbf{s}^A \cdot \mathbf{e}_i \rangle = \frac{3}{\alpha_x} \langle \mathbf{x} \cdot \mathbf{e}_i \rangle$$

$$\bar{B}_i = \langle \mathbf{s}^B \cdot \mathbf{e}_i \rangle = \frac{3}{\alpha_y} \langle \mathbf{y} \cdot \mathbf{e}_i \rangle$$

$$C_{ij} = \langle (\mathbf{s}^A \cdot \mathbf{e}_i)(\mathbf{s}^B \cdot \mathbf{e}_j) \rangle = \frac{9}{\alpha_x \alpha_y} \langle (\mathbf{x} \cdot \mathbf{e}_i)(\mathbf{y} \cdot \mathbf{e}_j) \rangle$$

observable

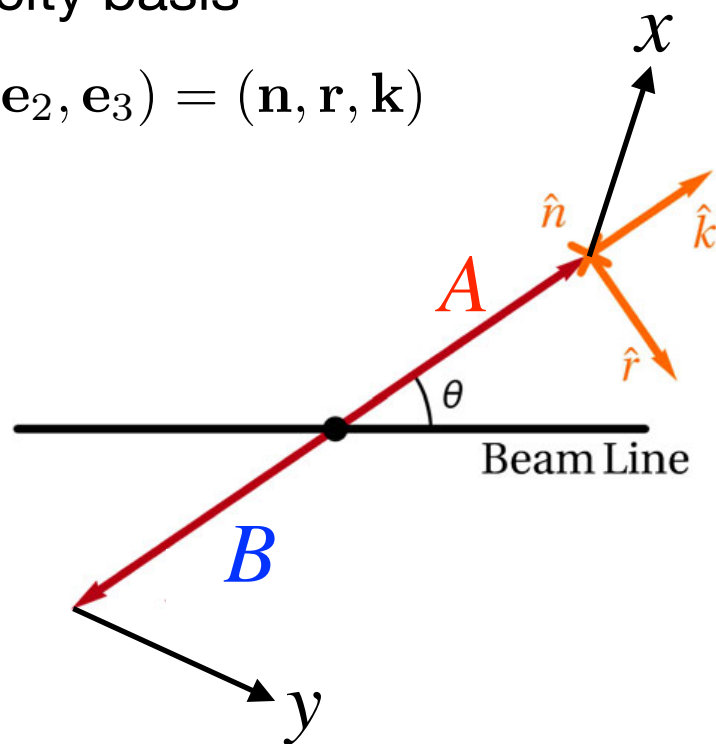
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neutrino momenta must be reconstructed (by solving kinematics) to reconstruct the top and antitop rest frames.

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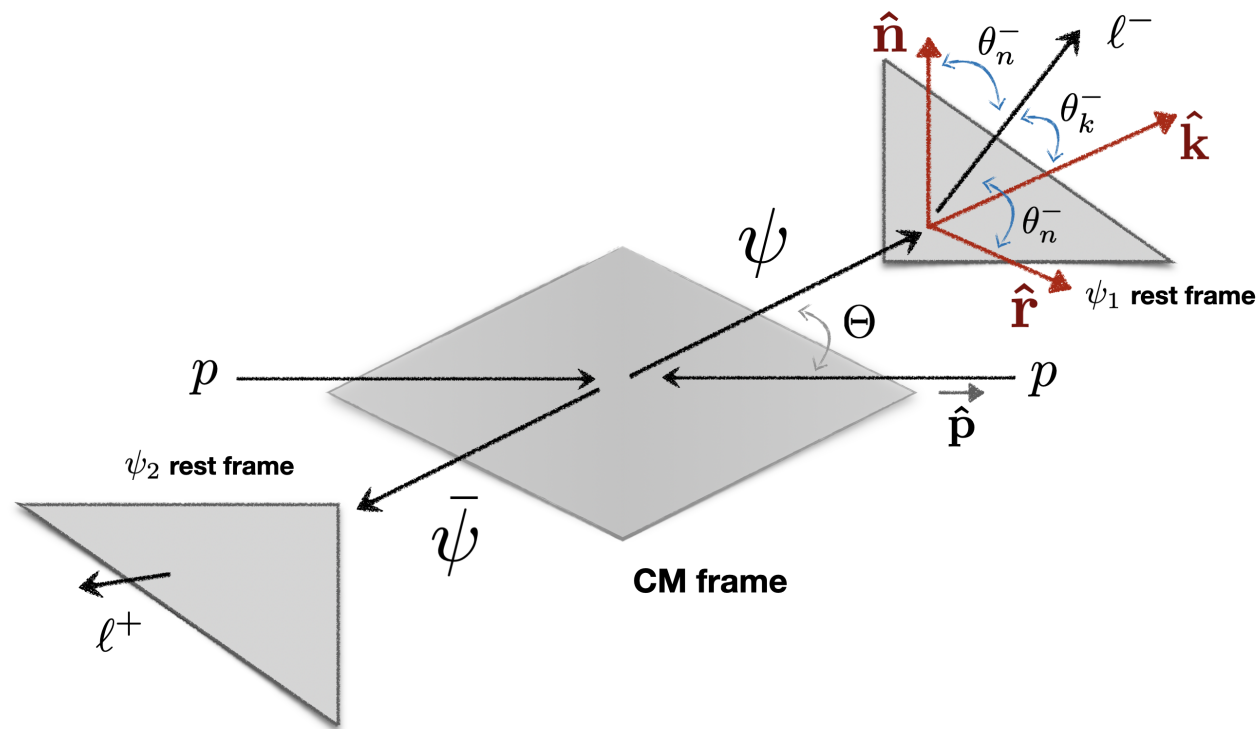
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[A.J.Barr, M.Fabbrichesi, R.Floresanini,
E.Gabrielli, L.Marzola 2402.07972]

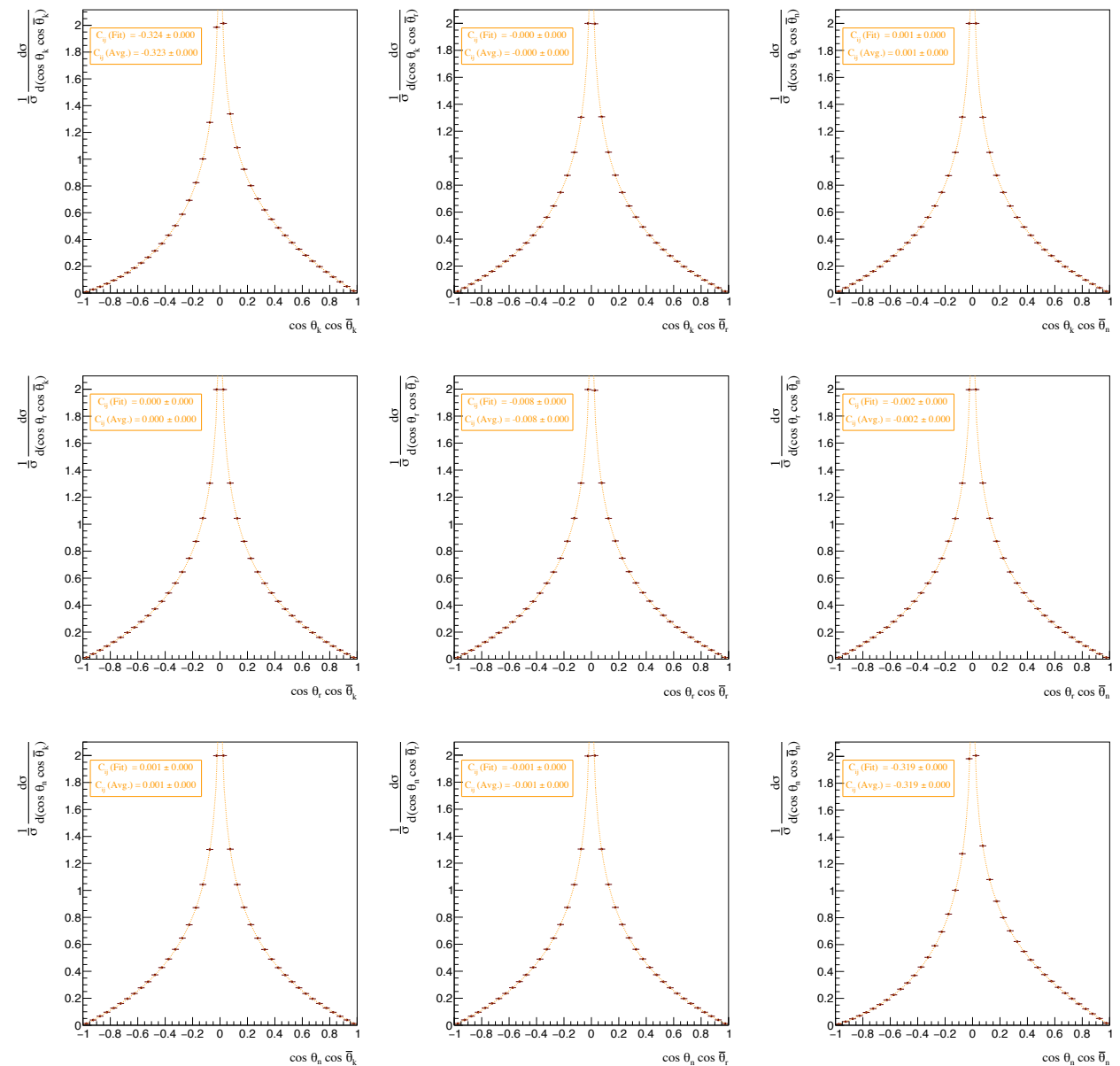
$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^\pm} = \frac{1}{2} (1 \mp B_i^\pm \cos \theta_i^\pm)$$

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^+ d \cos \theta_j^-} = \frac{1}{4} (1 + C_{ij} \cos \theta_i^+ \cos \theta_j^-)$$

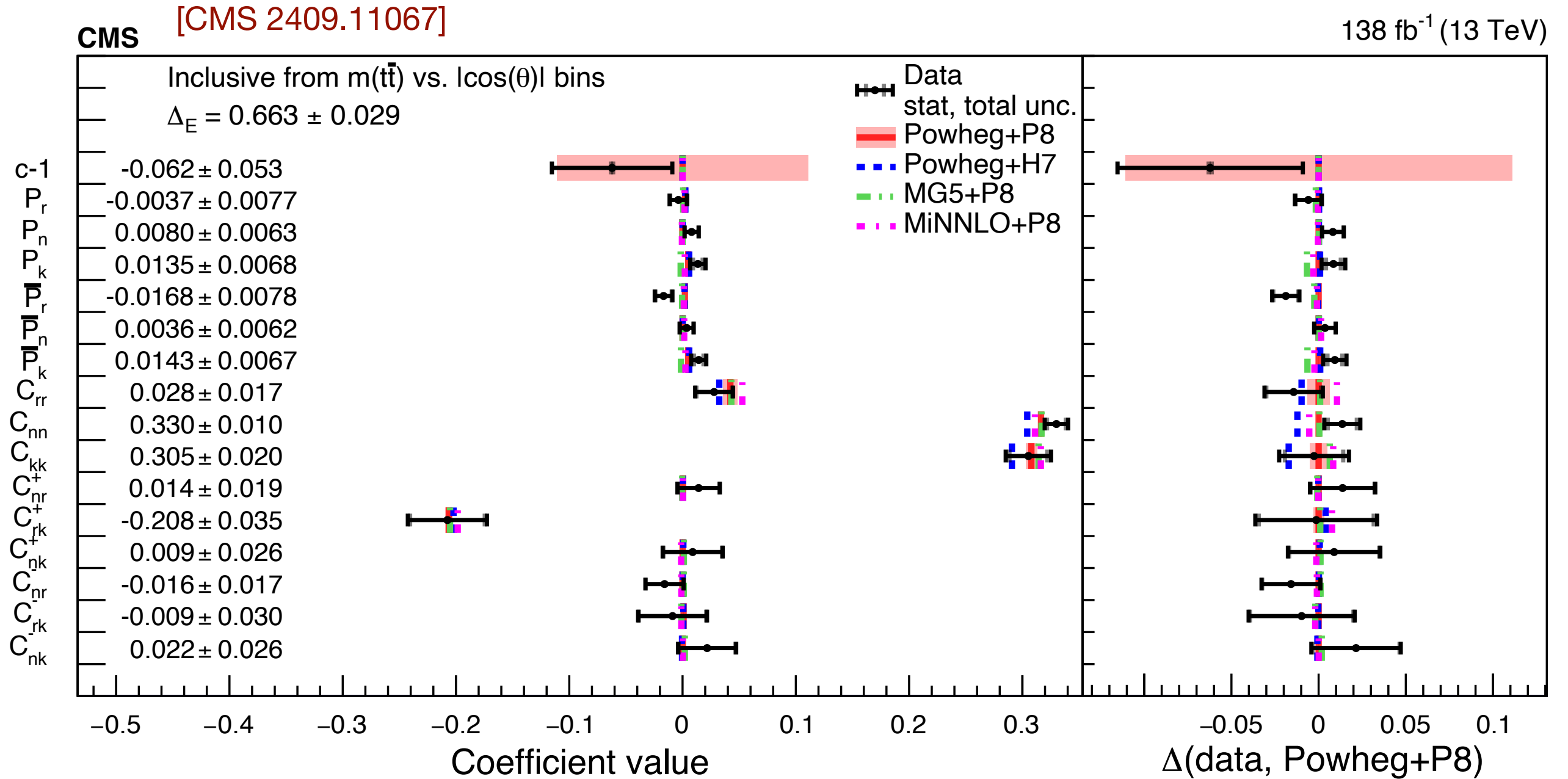
$$\rho = \frac{1}{4} \left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \sum_{i=1}^3 B_i^+ (\sigma_i \otimes \mathbf{1}_2) + \sum_{i=1}^3 B_i^- (\mathbf{1}_2 \otimes \sigma_i) + \sum_{i,j=1}^3 C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

Alternatively, one can fit the angular distributions to obtain $B_i^\pm, C_{ij}$

[Claudio Severi]

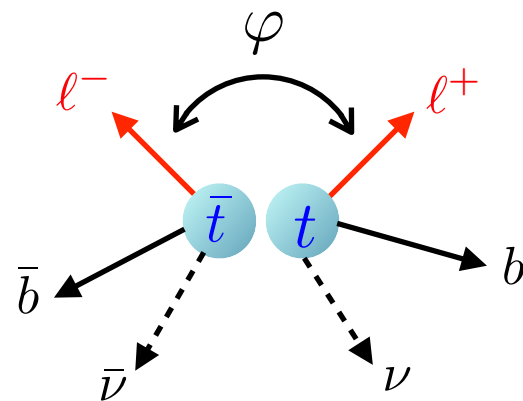
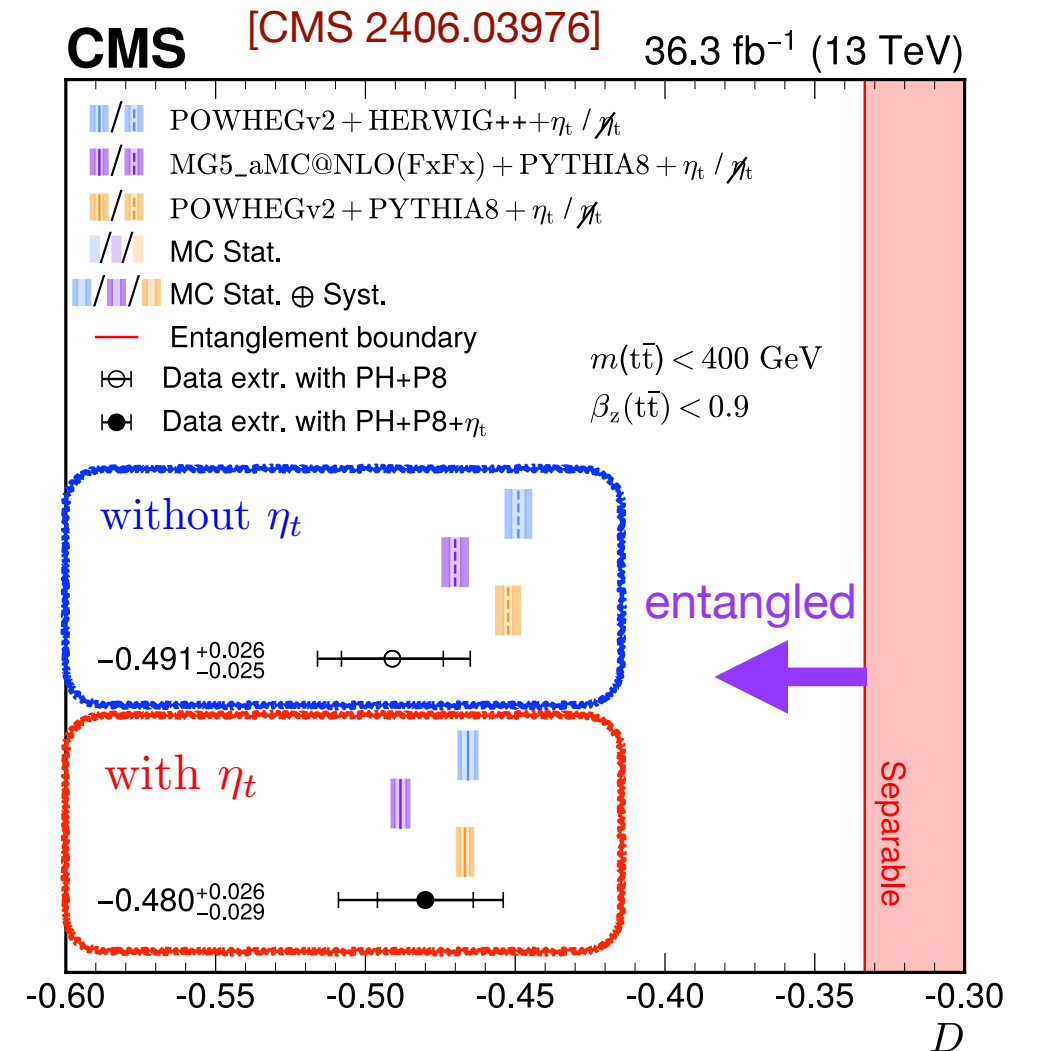
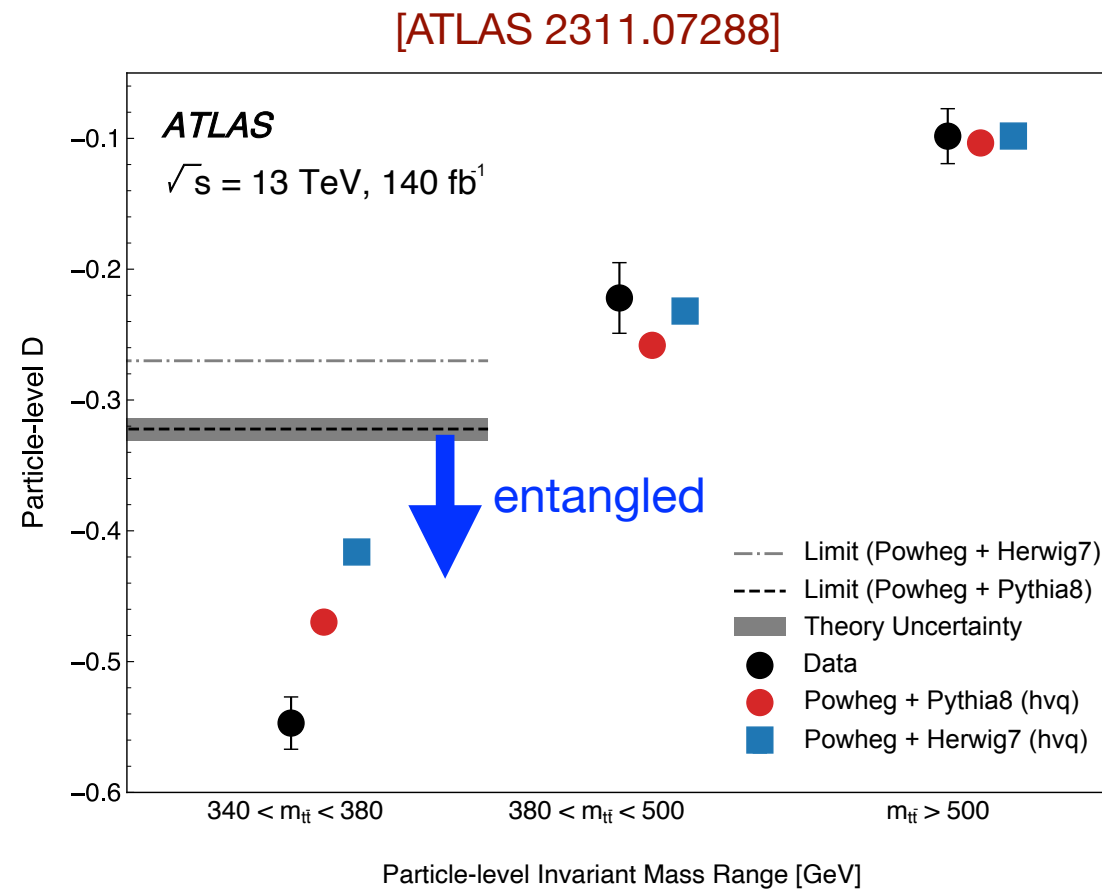


Quantum State Tomography of the $t\bar{t}$ spin state at LHC



$$\rho = \frac{1}{4} \left(\mathbf{1}_{t\bar{t}} + P_i \cdot [\sigma^i \otimes \mathbf{1}_{\bar{t}}] + \bar{P}_i \cdot [\mathbf{1}_t \otimes \sigma^i] + C_{ij} \cdot [\sigma^i \otimes \sigma^j] \right)$$

The highest-energy observation of entanglement!



$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\varphi} = \frac{1}{2} (1 - D \cos\varphi)$$

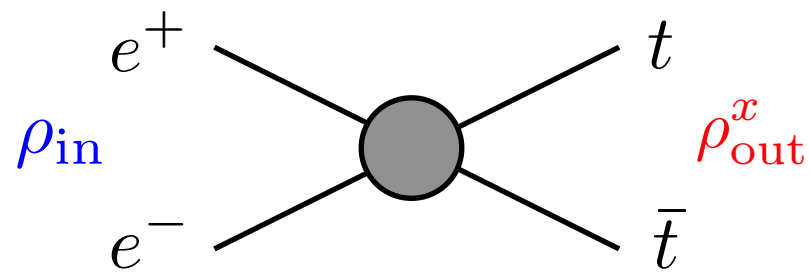
$$D \equiv \frac{\text{Tr}[C]}{3} < -\frac{1}{3}$$

is a sufficient condition for entanglement [Y.Afik, J.R.M.Nova 2003.02280]

Quantum *Process* Tomography of $e^+e^- \rightarrow t\bar{t}$ with polarised lepton colliders

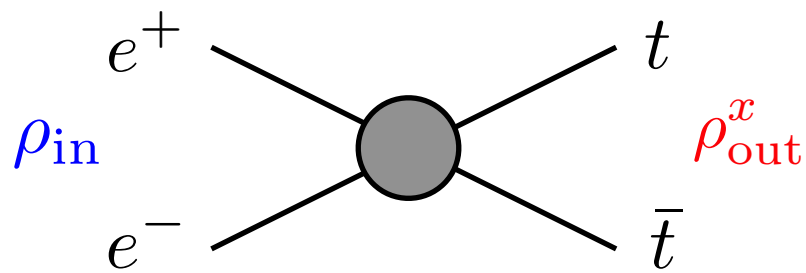
Clelia Altomonte, Alan J. Barr, Michał Eckstein, Paweł Horodecki, KS

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What we do:

- collide e^+e^- with a certain initial spin state, ρ_{in} ← collision = unitary time evolution by $\hat{S}$
- observe the $t\bar{t}$ final state in a certain momentum region, x ← measurement
- reconstruct the $t\bar{t}$ spin state ρ_{out}^x



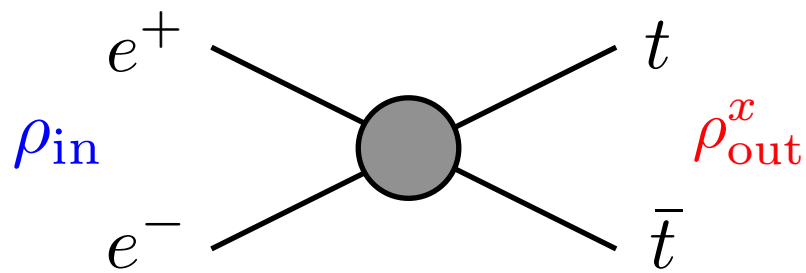
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Mathematical formulation:

- for each x , the above procedure defines a function (or a map), $\mathcal{I}_x$, that takes ρ_{in} to ρ_{out}^x

$$\mathcal{I}_x(\rho_{\text{in}}) = \rho_{\text{out}}^x$$



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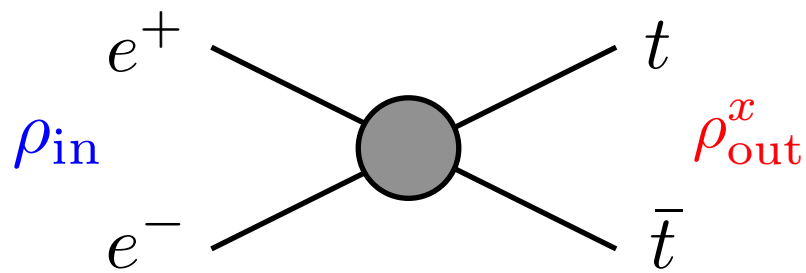
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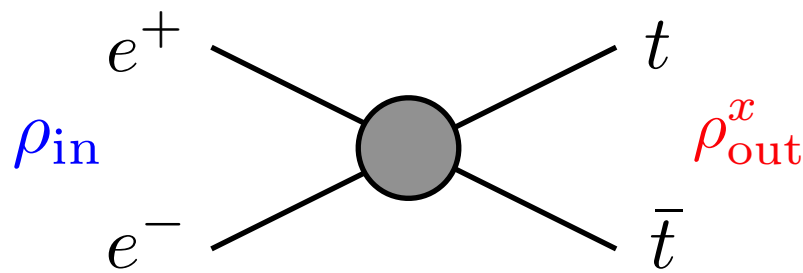
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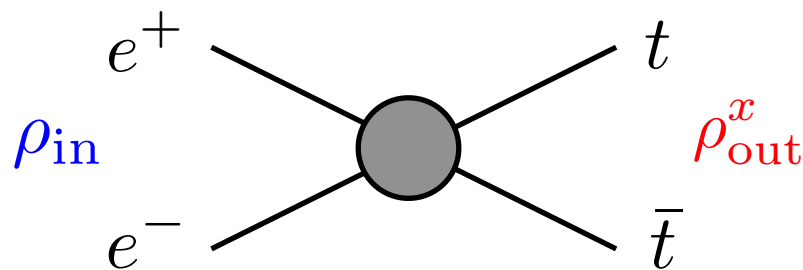
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- through $\mathcal{I}_x$, we can calculate the output ρ_{out}^x for any input ρ_{in} , meaning $\mathcal{I}_x$ is the *theory itself!*
- directly reconstruct $\mathcal{I}_x$ using polarised lepton colliders $\Rightarrow$ “*direct reconstruction* of the *theory*”!



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- reconstruct the $t\bar{t}$ spin state ρ_{out}^x

Mathematical formulation:

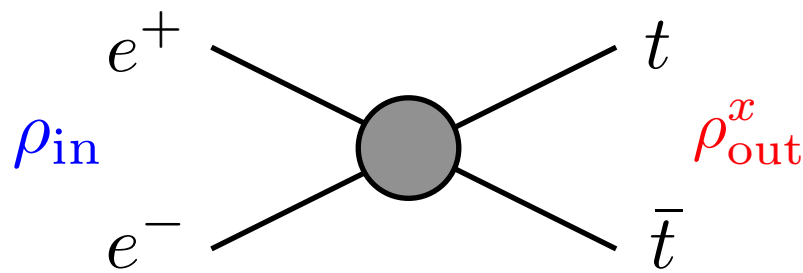
- for each x , the above procedure defines a function (or a map), $\mathcal{I}_x$, that takes ρ_{in} to ρ_{out}^x

$$\mathcal{I}_x(\rho_{\text{in}}) = \rho_{\text{out}}^x$$

- understand the conditions of $\mathcal{I}_x$, which must be satisfied in any quantum theories.

⇒ If one of the conditions is violated, QM will be falsified!

- through $\mathcal{I}_x$, we can calculate the output ρ_{out}^x for any input ρ_{in} , meaning $\mathcal{I}_x$ is the *theory itself!*
- directly reconstruct $\mathcal{I}_x$ using polarised lepton colliders ⇒ “direct reconstruction of the *theory*”!
- provides a new framework for the SM test, as well as the test of the QM itself!



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Quantum Instrument

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Quantum Instrument

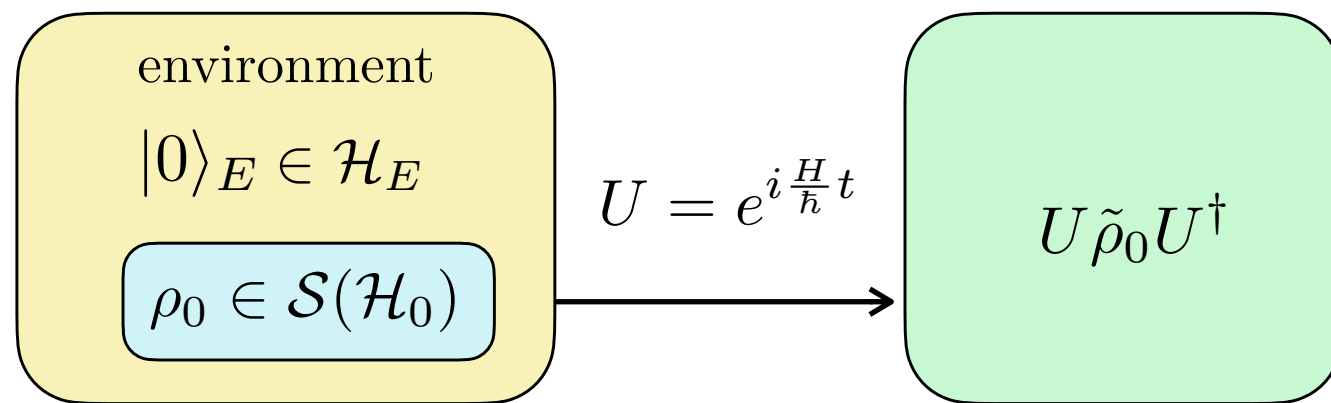
environment

$$|0\rangle_E \in \mathcal{H}_E$$

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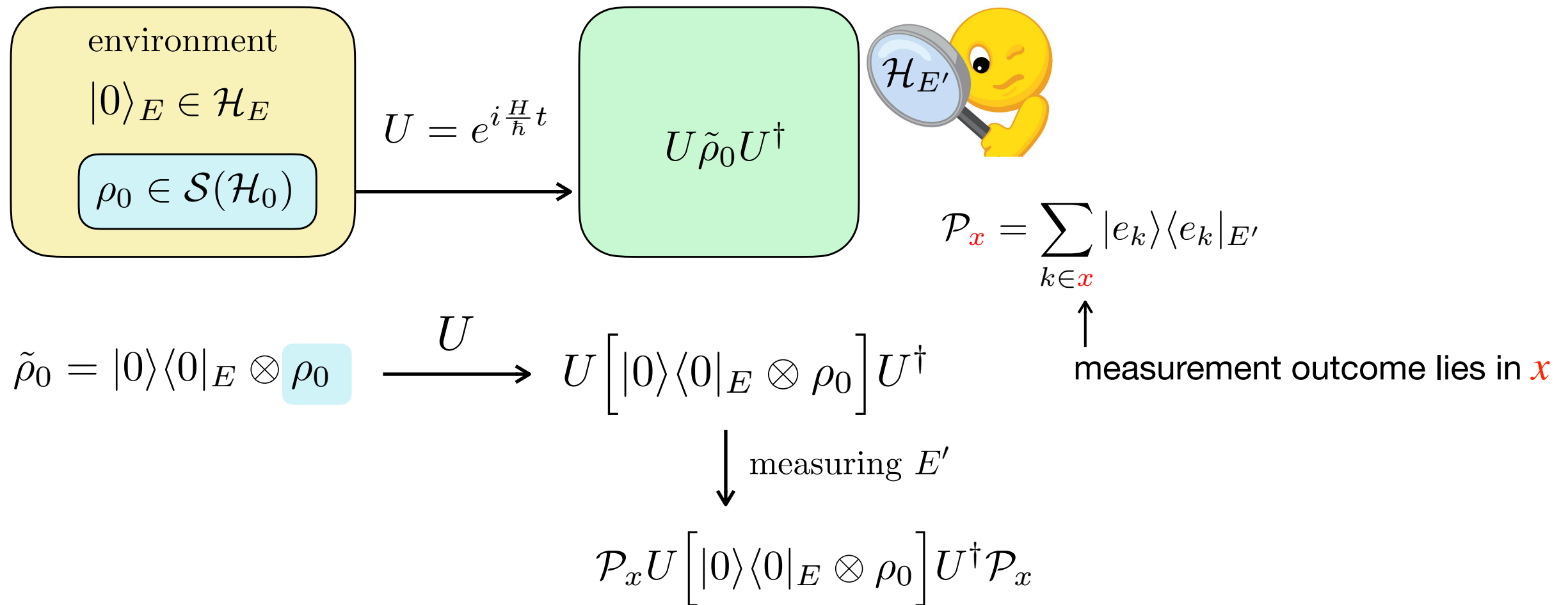
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Quantum Instrument

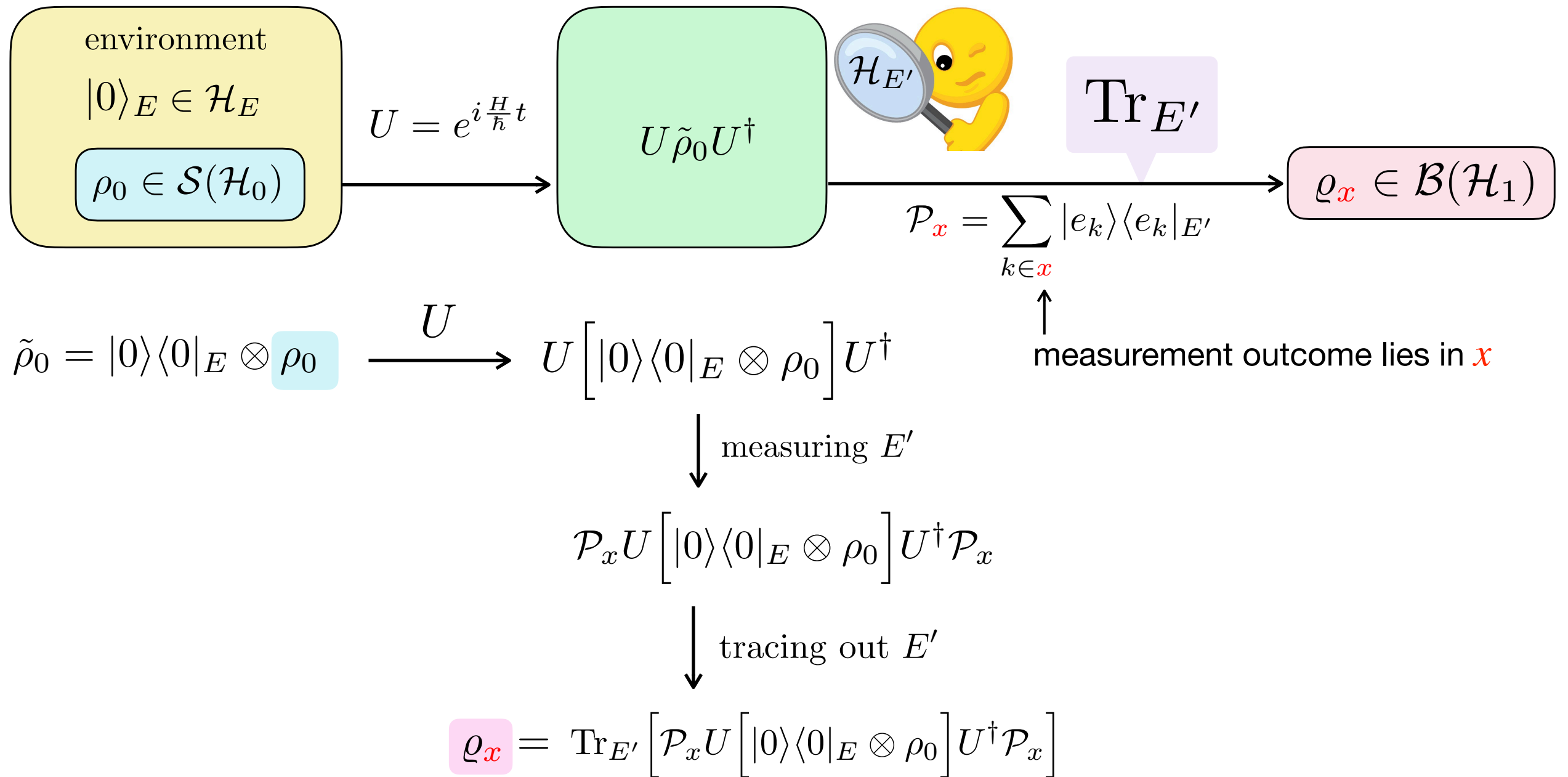


$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{U} U \left[|0\rangle\langle 0|_E \otimes \rho_0 \right] U^\dagger$$

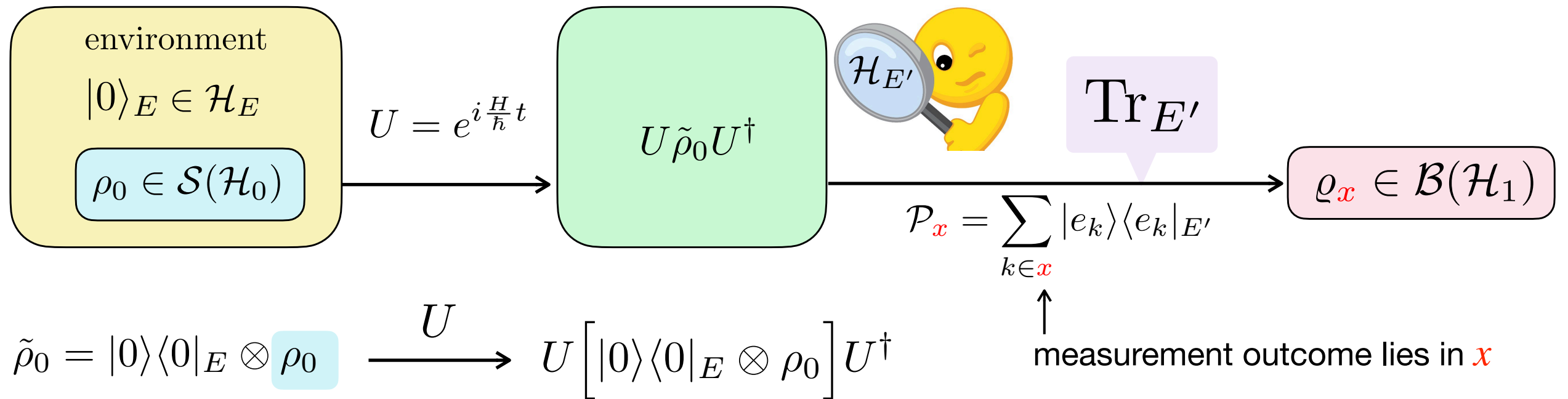
Quantum Instrument



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Quantum Instrument



Kraus operators:

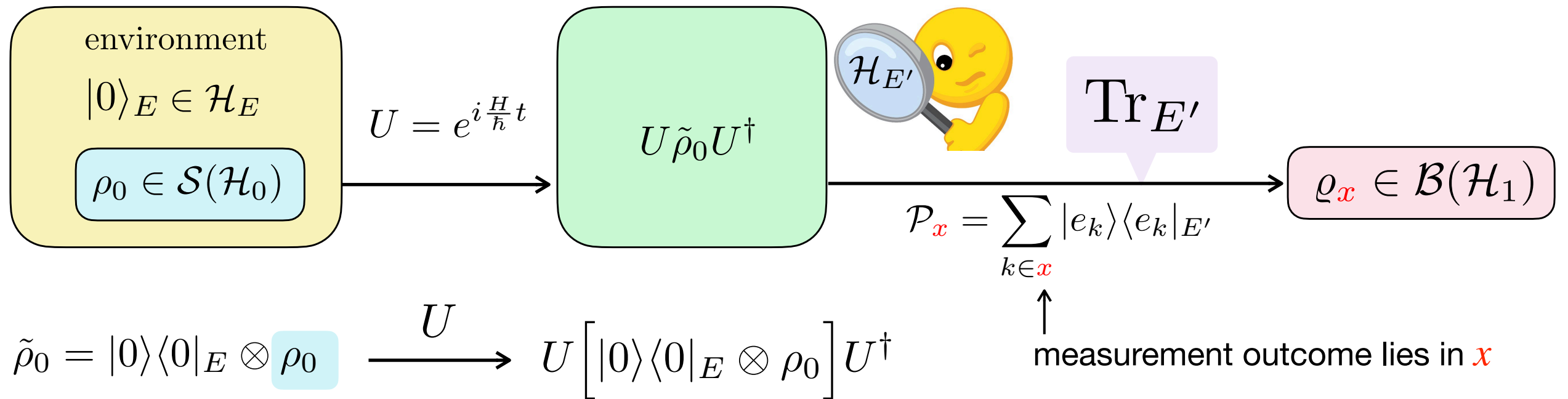
$$E_k = \langle e'_k | U | 0 \rangle_E$$

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$$\sum_{k \in x} E_k^\dagger E_k \leq 1$$

$$\begin{aligned} \rho_x &= \text{Tr}_{E'} \left[\mathcal{P}_x U \left[|0\rangle\langle 0|_E \otimes \rho_0 \right] U^\dagger \mathcal{P}_x \right] \\ &= \sum_{k \in x} \langle e'_k | U | 0 \rangle_E \langle 0|_E \otimes \rho_0 U^\dagger | e'_k \rangle \\ &= \sum_{k \in x} E_k \rho_0 E_k^\dagger \end{aligned}$$

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Quantum
Instrument I_x

$$I_x : \mathcal{S}(\mathcal{H}_0) \rightarrow \mathcal{B}(\mathcal{H}_1)$$

$$I_x(\rho_0) = \rho_x$$

- In any quantum theories, the map $\rho_{\text{in}} \rightarrow \rho_x$ is given by the **quantum instrument**:

$$\mathcal{I}_x(\rho_{\text{in}}) = \rho_x = \sum_{k \in x} E_k \rho_{\text{in}} E_k^\dagger$$

- The properties of the quantum instrument:

- **Linear**

- **Positive** $\rho_x = \sum_{k \in x} (E_k \sqrt{\rho_{\text{in}}})(E_k \sqrt{\rho_{\text{in}}})^\dagger \geq 0$

$\Rightarrow$ If one of the conditions is violated, **QM will be falsified!**

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$$\sum_{k \in x} E_k^\dagger E_k \leq 1$$

Outcome ρ_x is **not** normalised:

post-QI (normalised) **state**

$$\rho_x = \frac{\rho_x}{\text{Tr} \rho_x}$$

$\leftarrow$ state-to-(normalised)state map is **NOT** linear

Choi matrix

- $\mathcal{I}_x$ is a **linear** transition from a 4×4 matrix ρ^{in} (e^+e^- spin state) to a 4×4 matrix ρ_x ($t\bar{t}$ spin state)
- Such a linear function can be represented by a 16×16 matrix, **the Choi matrix**

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$$\hat{\rho}_{\text{in}} = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,L]}^{\text{in}} \underbrace{|I, J\rangle\langle K, L|}_{4 \times 4 = 16 \text{ basis operators in } \mathcal{H}_{\text{in}}}$$

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 &\quad \text{by the } t\bar{t} \text{ basis: } |A, B\rangle = |A\rangle_t \otimes |B\rangle_{\bar{t}} \\
 &\quad A, B, C, D \in \{0, 1\}
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 $d_{\text{in}} = \dim \mathcal{H}_{\text{in}}$

$$\tilde{\mathcal{I}}_x(|I, J\rangle\langle K, L|)_{[A,B],[C,D]} = \frac{1}{\dim \mathcal{H}_{\text{in}}} \begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|)_{[A,B],[C,D]} & \mathcal{I}_x(|++\rangle\langle+-|)_{[A,B],[C,D]} & \dots \\ \mathcal{I}_x(|-+\rangle\langle++|)_{[A,B],[C,D]} & \dots & \\ \mathcal{I}_x(|+-\rangle\langle++|)_{[A,B],[C,D]} & \dots & \\ \mathcal{I}_x(|--\rangle\langle++|)_{[A,B],[C,D]} & \dots & \mathcal{I}_x(|--\rangle\langle--|)_{[A,B],[C,D]} \end{pmatrix}$$

Quantum Process Tomography

- The Choi matrix, $\tilde{\mathcal{I}}_x$, has the entire information about the QI map, $\mathcal{I}_x : \rho_{\text{in}} \rightarrow \rho_{\text{out}}$
- In some sense, it is a “*matrix representation of the theory*”
- Different theories must give different Choi matrices
- Experimental reconstruction of Choi matrix is called **Q**uantum **P**rocess **T**omography (**QPT**)

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- Experimental reconstruction of Choi matrix is called **Quantum Process Tomography (QPT)**

- **QPT offers:**

- Powerful **SM test**: $[\tilde{\mathcal{I}}_x]^{\text{exp}} \Leftrightarrow [\tilde{\mathcal{I}}_x]^{\text{SM}}$

- **Positivity** test of the QM:

QM is excluded if one of the eigenvalues of $[\tilde{\mathcal{I}}_x]^{\text{exp}}$ is negative

- **Linearity** test of the QM:

QM is be excluded if

an initial state independent from the ones used in the QPT

$$[\varrho_x]^{\text{exp}}_{[A,B][C,D]} \neq \sum_{[I,J][K,L]} \rho_{[I,J][K,L]}^{\text{in}} [\tilde{\mathcal{I}}_x(|I,J\rangle\langle K,L|)]^{\text{exp}}_{[A,B][C,D]}$$


$$e^- e^+ \rightarrow t \bar{t}$$

Choi matrix in QFT

$$\rho_0 = \sum_{s_e} q_{s_e} |s_e\rangle \langle s_e| \quad s_e = \{++, +-, -+, --\} \quad \left(q_{s_e} \geq 0, \sum_{s_e} q_{s_e} = 1 \right)$$

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$t\bar{t}$ phase-space selection

$$\mathcal{P}_x = \sum_{s_t} \int_x d\Pi_{t\bar{t}} |p_t, s_t\rangle \langle p_t, s_t|$$

$$\hat{1} = \sum_f \left[\left(\prod_{i \in f} \int d\Pi_i \right) |f\rangle \langle f| \right], \quad d\Pi_i = \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i}$$

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$$\rho_0^{\text{mix}} = \frac{1}{4} \sum_{s_e} |s_e\rangle \langle s_e| \quad \propto \frac{1}{\sigma_{\mathcal{N}}} \sum_{s_e, s_t, s'_t} q_{s_e} \left[\frac{1}{2s} \int_x d\Pi_{\text{LIPS}}^{t\bar{t}} \mathcal{M}_{p_t, s_t}^{p_{\text{in}}, s_e} [\mathcal{M}_{p_t, s'_t}^{p_{\text{in}}, s_e}]^* \right] |s_t\rangle \langle s'_t|$$

$$\sigma_{\mathcal{N}} = \sigma[e^- e^+ (\rho_0^{\text{mix}}) \rightarrow t\bar{t}]$$

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$$\sigma_{\mathcal{N}} = \sigma[e^- e^+ (\rho_0^{\text{mix}}) \rightarrow t\bar{t}]$$

$$E(p_f) = \frac{1}{\sqrt{2s\sigma_{\mathcal{N}}}} \sum_{I, J, A, B} \mathcal{M}_{A, B}^{I, J} |A, B\rangle \langle I, J|$$

QFT formula for the Choi matrix:

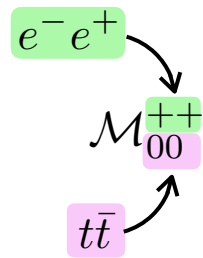
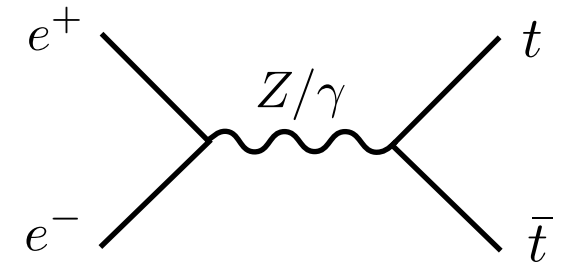
$$d\tilde{\mathcal{I}}(|I, J\rangle \langle K, L|)_{(A, B), (C, D)} = \frac{1}{\sigma_{\mathcal{N}}} \frac{1}{2s} \int_x d\Pi \mathcal{M}_{A, B}^{I, J} (\mathcal{M}_{C, D}^{K, L})^*$$

Standard Model prediction

- We compute the **Choi matrix** in the **SM** at **LO**:

$$\mathcal{L} \ni \sum_i \frac{1}{\Lambda_i^2} [\bar{\psi}_e \gamma_\mu (c_L^i P_L + c_R^i P_R) \psi_e] [\bar{\psi}_t \gamma^\mu (d_L^i P_L + d_R^i P_R) \psi_t]$$

i	Λ_i^2	c_L^i	c_R^i	d_L^i	d_R^i
A	s	$-e$	$-e$	$\frac{2}{3}e$	$\frac{2}{3}e$
Z	$s - m_Z^2$	$g_Z \left(-\frac{1}{2} + \sin^2 \theta_w\right)$	$g_Z \sin^2 \theta_w$	$g_Z \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w\right)$	$g_Z \left(-\frac{2}{3} \sin^2 \theta_w\right)$



$$\begin{aligned} \mathcal{M}_{00}^{++} &= \mathcal{M}_{11}^{++} = e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} \gamma^{-1} c_R^i \sin \theta (d_L^i + d_R^i), \\ \mathcal{M}_{01}^{++} &= -e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_R^i (1 + \cos \theta) [d_L^i (1 - \beta) + d_R^i (1 + \beta)], \\ \mathcal{M}_{10}^{++} &= -e^{i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_R^i (1 - \cos \theta) [d_L^i (1 + \beta) + d_R^i (1 - \beta)], \\ \mathcal{M}_{00}^{--} &= \mathcal{M}_{11}^{--} = -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} \gamma^{-1} c_L^i \sin \theta (d_L^i + d_R^i), \\ \mathcal{M}_{01}^{--} &= -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_L^i (1 - \cos \theta) [d_L^i (1 - \beta) + d_R^i (1 + \beta)], \\ \mathcal{M}_{10}^{--} &= -e^{-i\phi} \sum_i \frac{s}{2\Lambda_i^2} c_L^i (1 + \cos \theta) [d_L^i (1 + \beta) + d_R^i (1 - \beta)], \end{aligned}$$

$$\mathcal{M}_{A,B}^{+-} = \mathcal{M}_{A,B}^{-+} = 0$$

$$\sigma_N = \frac{N_C}{128\pi^2} \frac{q}{s\sqrt{s}} \int \sum_{I,J,A,B} |\mathcal{M}_{A,B}^{I,J}|^2 d\Omega$$

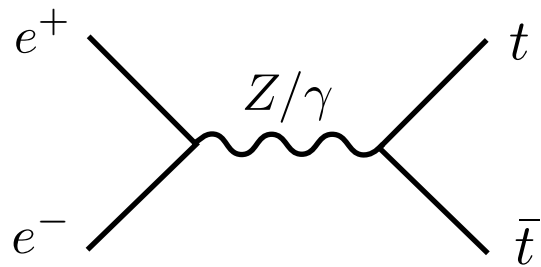
$$d\tilde{\mathcal{I}}(|I, J\rangle\langle K, L|)_{(A,B),(C,D)} = \frac{1}{\sigma_N} \frac{1}{2s} \int_x d\Pi \mathcal{M}_{A,B}^{I,J} (\mathcal{M}_{C,D}^{K,L})^*$$

$$\tilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++ , ++)} & 0 & 0 & I_x^{(++ , --)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(-- , ++)} & 0 & 0 & I_x^{(-- , --)} \end{pmatrix} \quad I_x^{(-- , ++)} = [I_x^{(++ , --)}]^\dagger$$

$$\begin{aligned} \frac{dI^{(++ , ++)}}{d \cos \theta d\phi} &= \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta (1 + c_\theta) & a_{13}^{(+)} s_\theta (1 - c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta (1 + c_\theta) & a_{22}^{(+)} (1 + c_\theta)^2 & a_{23}^{(+)} (1 - c_\theta^2) & a_{24}^{(+)} s_\theta (1 + c_\theta) \\ a_{31}^{(+)} s_\theta (1 - c_\theta) & a_{32}^{(+)} (1 - c_\theta^2) & a_{33}^{(+)} (1 - c_\theta)^2 & a_{34}^{(+)} s_\theta (1 - c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta (1 + c_\theta) & a_{43}^{(+)} s_\theta (1 - c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix}, \\ \frac{dI^{(++ , --)}}{d \cos \theta d\phi} &= e^{i2\phi} \begin{pmatrix} a_{11}^{(+-)} s_\theta^2 & a_{12}^{(+-)} s_\theta (1 - c_\theta) & a_{13}^{(+-)} s_\theta (1 + c_\theta) & a_{14}^{(+-)} s_\theta^2 \\ a_{21}^{(+-)} s_\theta (1 + c_\theta) & a_{22}^{(+-)} (1 - c_\theta^2) & a_{23}^{(+-)} (1 + c_\theta)^2 & a_{24}^{(+-)} s_\theta (1 + c_\theta) \\ a_{31}^{(+-)} s_\theta (1 - c_\theta) & a_{32}^{(+-)} (1 - c_\theta)^2 & a_{33}^{(+-)} (1 - c_\theta^2) & a_{34}^{(+-)} s_\theta (1 - c_\theta) \\ a_{41}^{(+-)} s_\theta^2 & a_{42}^{(+-)} s_\theta (1 - c_\theta) & a_{43}^{(+-)} s_\theta (1 + c_\theta) & a_{44}^{(+-)} s_\theta^2 \end{pmatrix} \\ \frac{dI^{(-- , --)}}{d \cos \theta d\phi} &= \begin{pmatrix} a_{11}^{(-)} s_\theta^2 & a_{12}^{(-)} s_\theta (1 - c_\theta) & a_{13}^{(-)} s_\theta (1 + c_\theta) & a_{14}^{(-)} s_\theta^2 \\ a_{21}^{(-)} s_\theta (1 - c_\theta) & a_{22}^{(-)} (1 - c_\theta)^2 & a_{23}^{(-)} (1 - c_\theta^2) & a_{24}^{(-)} s_\theta (1 - c_\theta) \\ a_{31}^{(-)} s_\theta (1 + c_\theta) & a_{32}^{(-)} (1 - c_\theta^2) & a_{33}^{(-)} (1 + c_\theta)^2 & a_{34}^{(-)} s_\theta (1 + c_\theta) \\ a_{41}^{(-)} s_\theta^2 & a_{42}^{(-)} s_\theta (1 - c_\theta) & a_{43}^{(-)} s_\theta (1 + c_\theta) & a_{44}^{(-)} s_\theta^2 \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} a^{(+)}|_{\sqrt{s}=370 \text{ GeV}} &= \begin{pmatrix} 0.503 & -1.442 & 0.364 & 0.503 \\ -1.442 & 4.137 & -1.043 & -1.442 \\ 0.364 & -1.043 & 0.263 & 0.364 \\ 0.503 & -1.442 & 0.364 & 0.503 \end{pmatrix} \cdot 10^{-2}, & a^{(+)}|_{\sqrt{s}=1 \text{ TeV}} &= \begin{pmatrix} 0.219 & -0.777 & -0.494 & 0.219 \\ -0.777 & 2.755 & 1.751 & -0.777 \\ -0.494 & 1.751 & 1.113 & -0.494 \\ 0.219 & -0.777 & -0.494 & 0.219 \end{pmatrix} \cdot 10^{-2}, \\ a^{(+-)}|_{\sqrt{s}=370 \text{ GeV}} &= \begin{pmatrix} -0.800 & 0.264 & -1.979 & -0.800 \\ 2.293 & -0.758 & 5.676 & 2.293 \\ -0.578 & 0.191 & -1.431 & -0.578 \\ -0.800 & 0.264 & -1.980 & -0.800 \end{pmatrix} \cdot 10^{-2}, & a^{(+-)}|_{\sqrt{s}=1 \text{ TeV}} &= \begin{pmatrix} -0.340 & -0.810 & -1.162 & -0.340 \\ 1.205 & 2.870 & 4.117 & 1.205 \\ 0.766 & 1.824 & 2.617 & 0.766 \\ -0.340 & -0.810 & -1.162 & -0.340 \end{pmatrix} \cdot 10^{-2}, \\ a^{(-)}|_{\sqrt{s}=370 \text{ GeV}} &= \begin{pmatrix} 1.271 & -0.420 & 3.146 & 1.271 \\ -0.420 & 0.139 & -1.040 & -0.420 \\ 3.146 & -1.040 & 7.788 & 3.146 \\ 1.271 & -0.420 & 3.146 & 1.271 \end{pmatrix} \cdot 10^{-2}, & a^{(-)}|_{\sqrt{s}=1 \text{ TeV}} &= \begin{pmatrix} 0.527 & 1.256 & 1.802 & 0.527 \\ 1.256 & 2.990 & 4.290 & 1.256 \\ 1.802 & 4.290 & 6.154 & 1.802 \\ 0.527 & 1.256 & 1.802 & 0.527 \end{pmatrix} \cdot 10^{-2}, \end{aligned}$$

This prediction should be compared with the **experiment**



$$\mathcal{M}_{A,B}^{+-} = \mathcal{M}_{A,B}^{-+} = 0$$

$$\tilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++ , ++)} & 0 & 0 & I_x^{(++ , --)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(-- , ++)} & 0 & 0 & I_x^{(-- , --)} \end{pmatrix}$$



The zero structure is unique for the **s-channel vector boson** exchange at **LO**

- This zero structure is broken by the **LO** contributions of various types of **new physics**:

s-channel scalar



$$\frac{c_S}{\Lambda^2} [\bar{\psi}_e \psi_e] [\bar{\psi}_t \psi_t],$$

s-channel tensor



$$\frac{c_T}{\Lambda^2} [\bar{\psi}_e \sigma_{\mu\nu} \psi_e] [\bar{\psi}_t \sigma^{\mu\nu} \psi_t],$$

t-channel scalar



$$\frac{c_t}{\Lambda^2} [\bar{\psi}_e \psi_t] [\bar{\psi}_t \psi_e], \dots$$

- In the QPT approach, **LO** of **new physics** can directly competes with **NLO** of **SM**.

Powerful tool to search for new physics!

QPT with polarised lepton collider

- The density operator representing a polarised beam in the $\mathbf{n}$ direction with the degree $0 \leq q \leq 1$

$$\eta(q, \mathbf{n}) = q|\mathbf{n}\rangle\langle\mathbf{n}| + (1 - q)\mathbf{1} = \frac{1}{2} [(1 + q)|\mathbf{n}\rangle\langle\mathbf{n}| + (1 - q)|-\mathbf{n}\rangle\langle-\mathbf{n}|] = \sum_{I, K=\pm} C_{(q, \mathbf{n})}^{(I, K)} |I\rangle\langle K|$$

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- Consider 4 electron beam settings: $\rho_{\text{in}, e^-}^+ = \eta(q, +)$, $\rho_{\text{in}, e^-}^- = \eta(q, -)$, $\rho_{\text{in}, e^-}^{\vec{x}} = \eta(q, \vec{x})$, $\rho_{\text{in}, e^-}^{\vec{y}} = \eta(q, \vec{y})$
- Consider 4 positron beam settings: $\rho_{\text{in}, e^+}^+ = \eta(\bar{q}, +)$, $\rho_{\text{in}, e^+}^- = \eta(\bar{q}, -)$, $\rho_{\text{in}, e^+}^{\vec{x}} = \eta(\bar{q}, \vec{x})$, $\rho_{\text{in}, e^+}^{\vec{y}} = \eta(\bar{q}, \vec{y})$

we need transversely
polarised beams



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$$|I\rangle\langle K| = \sum_a X_a^{(I,K)} \rho_{e-}^a \quad |J\rangle\langle L| = \sum_b Y_b^{(I,K)} \rho_{e+}^b$$

$$X = \frac{1}{2q} \begin{pmatrix} 1+q & -(1-q) & 0 & 0 \\ -(1+i) & -(1+i) & 2 & 2i \\ -1+i & -1+i & 2 & -2i \\ -(1-q) & 1+q & 0 & 0 \end{pmatrix} \quad Y = X|_{q \rightarrow \bar{q}}$$

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- The basis operators are related with the initial state as $|I, J\rangle\langle K, L| = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} \rho_{\text{in}}^{(a,b)}$

- The Choi matrix is obtained as: $\mathcal{I}_x(|I, J\rangle\langle K, L|) = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} \mathcal{I}_x(\rho_{\text{in}}^{(a,b)}) = \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} \rho_x^{(a,b)}$

$$\tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A, B], [C, D]} = \frac{1}{4} \sum_{a, b} X_a^{(I, K)} Y_b^{(J, L)} [\varrho_x^{(a, b)}]_{[A, B], [C, D]}$$

$$\tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A,B],[C,D]} = \frac{1}{4} \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} [\varrho_x^{(a,b)}]_{[A,B],[C,D]}$$



$$\tilde{\mathcal{I}}_x^{\text{SM}} = \begin{pmatrix} I_x^{(++++)} & 0 & 0 & I_x^{(++--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--++)} & 0 & 0 & I_x^{(----)} \end{pmatrix}$$

Precision test of SM

$$\tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A, B], [C, D]} = \frac{1}{4} \sum_{a, b} X_a^{(I, K)} Y_b^{(J, L)} [\varrho_x^{(a, b)}]_{[A, B], [C, D]}$$

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Precision test of SM

$$\lambda_{\min}(\tilde{\mathcal{I}}_x) = \min \{ \lambda \mid \det(\tilde{\mathcal{I}}_x - \lambda) = 0 \} \geq 0 ?$$

Positivity test of QM

$$\tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A,B],[C,D]} = \frac{1}{4} \sum_{a,b} X_a^{(I,K)} Y_b^{(J,L)} [\varrho_x^{(a,b)}]_{[A,B],[C,D]}$$

$$\tilde{\mathcal{I}}_x^{\text{SM}} = \begin{pmatrix} I_x^{(++++)} & 0 & 0 & I_x^{(++--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--++)} & 0 & 0 & I_x^{(----)} \end{pmatrix}$$

Precision test of SM

$$\lambda_{\min}(\tilde{\mathcal{I}}_x) = \min \{ \lambda \mid \det(\tilde{\mathcal{I}}_x - \lambda) = 0 \} \geq 0 ?$$

Positivity test of QM

$$[\varrho_x]_{[A,B],[C,D]}^{\text{prediction}} = \sum_{[I,J],[K,L]} \rho_{[I,J],[K,L]}^{\text{in,new}} \tilde{\mathcal{I}}_x^{\text{exp}}(|I, J\rangle\langle K, L|)_{[A,B],[C,D]}$$

check
experimentally

Linearity test of QM

Summary

- We formulated **spin-to-spin transitions** in particle collisions as **Quantum Instrument** — an quantum evolution of subsystem in an environment (environment = momentum Hilbert space).
- The state-to-state map can be described by **Choi matrix**, enabling us to write a theory in a matrix form: a 16 x 16 matrix for 2-qubit $\rightarrow$ 2-qubit
- Experimental reconstruction of the Choi matrix (**Quantum Process Tomography**) offers :

► A powerful probe of BSM physics

Two theories that predicting different spin-spin transition necessarily give different Choi matrices

► Foundational tests of Quantum Mechanics

- **Linearity test:** confront the Choi matrix prediction with the measurement
- **Positivity test:** eigenvalues of the Choi matrix must be non-negative

If one of these tests fails, **QM will be experimentally falsified!**

Thank you for listening!